Sun Block: Who Needs It?
Reading Assignment

Do you spend much time in the sun? On the beach, working in the garden, fishing, or playing soccer - many of our best hours are spent in the sunshine. Our bodies need the sun; we have to have it to get vitamin D, essential for good health. But too much exposure to the sun can be deadly.

The American Cancer Society (ACS) warns us that the number of cases of skin cancer is increasing. Skin cancer is usually caused by over-exposure to the sun's rays. Melanoma is said to be the most serious of several kinds of skin cancer because it has the lowest cure rate and must be detected even earlier than other skin cancers if it is to be cured. ACS estimated that there were 44,200 new cases of malignant melanoma in the U.S. in 1999. Skin cancers are curable if detected early but they can cause death if they remain untreated.

Our best protection from the sun’s rays is the ozone layer, but it has been seriously depleted by the production of chlorofluorocarbons here on earth (and other events). The less protection the ozone layer can give us, the more protection we need to provide ourselves.

The dark skin of people of African descent provides some protection from the dangers of the sun’s rays, but there is an increasing danger of skin cancers among African-Americans, as there is for all other groups. From 1962 to 1992, the death rate among African Americans due to skin cancer more than doubled. About 215 of every 100,000 African-American men died from skin cancer in 1992 and 118 of every 100,000 African-American women.

Geography is an important factor in skin cancer. In the tropics, where the midday sun beams down from directly overhead, we receive the most intense ultraviolet radiation. For this reason, a person in the tropics is at greater risk than the same person on a part of the earth where the rays of the sun meet the earth at a more oblique angle.

Dermatologists have found that even if there is no sunburn, over-exposure to the sun produces microscopic changes in the skin. They have found that sunblock is useful in preventing these changes, and thus, they believe, in preventing skin cancers that occur due to ultraviolet radiation.

African American dermatologists have recommended that people of African descent use a sunblock with SPF (sun protection factor) of 15 or higher. Generally, dermatologists have recommended that most others use an SPF of 30. What these SPF numbers mean is the subject of the next lesson.

0. What percent of African American men died from skin cancers in 1992? What percent of African American women?
Sun Block: Who Needs It?
A Mathematical Investigation

1. Discuss the reading assignment, “Sun Block: Who Needs It?” including question 0.

When Kayla learned about the dangers of skin cancer, she decided that she should find out about blocking out the sun with sun lotion. The lotions she saw were all labeled with SPF numbers, where SPF stands for “sun protection factor.” It was clear that the higher the SPF, the more protection you get. But Kayla’s doctor had told her that she needs to block at least 90% of the UV rays. How could she decide which SPF she needed? After some research, she found out that an SPF of 2 blocks 50% of the sun’s UVB rays, an SPF of 4 blocks 75% of the UVB rays, and an SPF of 6 blocks $83\frac{1}{3}$% of the UVB rays. (She also learned that protection from UVA rays is not measured by the SPF numbers.)

2. Complete the following input/output table using the information above.

<table>
<thead>
<tr>
<th>SPF</th>
<th>% of UVB rays blocked</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Is there a constant difference between the inputs? Is there a constant difference between the outputs?

4. We chose to use SPF as input. Why is that a good choice in the light of Kayla’s question?

5. Change the percents in your table to fractions and simplify all the fractions. Put the results in the following table.

<table>
<thead>
<tr>
<th>SPF</th>
<th>Fraction of UVB rays blocked</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Compare the SPF value to the denominator of the corresponding fraction.

7. Compare the numerator of each fraction to its denominator. Do you see a pattern?

8. If the pattern holds, what fraction would correspond to an SPF of 8?

   What percent of the sun’s UVB rays would be blocked by SPF 8?

9. What fraction would correspond to an SPF of 15?

   What percent of the sun’s UVB rays would be blocked by SPF 15?

10. What fraction would correspond to an SPF of 30?

    What percent of the sun’s UVB rays would be blocked by SPF 30?

11. What SPF would you recommend to Kayla so that she can take her doctor’s advice? Explain why.
Now you have answered Kayla’s question. You did so by discovering a pattern and using it. Let’s look at the situation again using graphs.

12. Graph the ordered pairs from your table in problem 5 and your answers to problems 8, 9 and 10 on your TI-83 using STAT Edit. Put all the SPF numbers in the L1 column and the corresponding fractions in the L2 column.

Then graph these ordered pairs by using (2nd) STAT PLOT. You must turn one of the Plots on the STAT PLOT menu to ON, choose the first Type of graph (it is called a scatterplot), put L1 in Xlist and L2 in Ylist.

One of the challenges of using a graphing calculator is setting the window well. For this graph, we suggest you use Xmin = 0, Xmax = 35, Ymin = 0, Ymax = 1. Either explain why these boundary choices are good choices, or use something else and explain why you think your choice is a better one.

13. Can you see a pattern on your graph? When the graph makes a nice pattern, we can usually find a nice algebraic equation for it. Study your table in problem 6. If the SPF is x, what is the denominator of the fraction? What is the numerator? Now go to the Y= screen. Type in the fraction, in terms of x. Put parentheses around the numerator, use $\div$ to make the fraction line, and then give the denominator. Graph.

You should see your original points from number 12 and the graph of this function you just put in. Describe what it looks like.

Teaching Guide for

Sun Block: Who Needs It?

Introduction: The mathematics in this lesson includes (1) the foundation of the function concept as an input/output mechanism, (2) pattern seeking, (3) development of an algebraic formula to generalize a pattern, (4) association of graphs and tables, and (5) the relationship between percents and fractions. The lesson can be used as an informal introduction to functions. It also introduces use of technology to produce plots of numerical data and graphs of algebraic functions. It is important that while students are doing this new and interesting mathematics they recognize the critical role played by their knowledge of fractions and percents in doing it. Use the lesson as an opportunity to strengthen their knowledge and skill in this area.

Students should work in groups of 3 or 4. Spend a little time at the opening of the lesson discussing group etiquette. Group members should speak out; they should not keep their ideas to themselves. At the same time, group members must listen respectfully to other members of the group, learn from each other, and help each other learn.

Answers and teaching suggestions:

0. 0.215% of men and 0.118% of women. These percents, while small, were rising and considered ominous by the American Cancer Society and the medical community.

1. Discussion might focus on human health and environmental issues. That we need mathematics to study these is an important lesson in itself. You may learn from students’ answers to question 0 that it will be necessary to review the percent concept.

Throughout these investigations, opportunities arise to assess students’ knowledge of elementary mathematics (including the arithmetic of percents and fractions). Take advantage of the knowledge you gain about students’ prerequisite knowledge and skills; provide the necessary experiences to help them overcome deficits both by giving clear explanations when opportunity arises and by assigning to individual students laboratory activities with concrete materials to help them overcome misconceptions.

2. Complete the following input/output table using the information above.

<table>
<thead>
<tr>
<th>SPF</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of UVB rays blocked</td>
<td>50</td>
<td>75</td>
<td>83 $\frac{1}{3}$</td>
</tr>
</tbody>
</table>

3. There is a constant difference between the inputs, but not between the outputs. If students have studied input/output tables for linear functions, this will alert them that the relationship we have here is not linear; the graph will not be a line.

4. We chose to use SPF as input because Kayla knew the SPF numbers and wanted to find out what percent of the rays were blocked by each one. SPF determined percent blocked, in her investigation.

5.
6. Comparing the SPF value to the denominator of the corresponding fraction, we see that they are the same number.

7. The numerator of each fraction is 1 less than its denominator.

8. Using the pattern we saw in #7, we expect an SPF of 8 to give us the fraction \( \frac{7}{8} \).

   From this we conclude that SPF 8 blocks 87.5% of the UVB rays.

9. \( \frac{14}{15} \); SPF 15 blocks 93\( \frac{1}{6} \)% of the UVB rays.

10. \( \frac{29}{30} \); 96\( \frac{2}{3} \)%

11. Kayla needs SPF 10 to block \( \frac{9}{10} \) = 90% of the rays. SPF 10 is hard to find, though.

   Students will usually recommend SPF 15. It is available, and they have just shown that it would be enough.

12. For most students, this will be a first experience with graphing discrete points on a graph. Be prepared to offer support and assistance. Doing it with them on your calculator, displayed on the overhead projector is helpful.

13. Students should describe the graphical pattern in their own words. They already saw the numerical pattern and used it to answer questions 6-11. Now they are asked to describe it algebraically as \( y = \frac{x - 1}{x} \). They should arrive at that if they answer the questions. If the SPF is \( x \), then the denominator of the fraction is \( x \). The numerator is 1 less, so it is \( x-1 \).

   Be sure students put parentheses around the numerator when they enter the fraction on the Y= screen. Do not graph the function on the overhead projector before your students graph it on their calculators. Watching students watch the graph of \( y = \frac{x - 1}{x} \) pass through the points they plotted, thereby seeing the connection between the numerical values that describe a relationship and the algebraic expression of that idea is one of the delights of teaching elementary algebra. Enjoy it!