Will Your Grandchildren Have Queen Conch?

Let x represent the number of some species of fish in a certain area. Let’s also pick a specified time interval, say 1 month. The species has a population growth rate per month, say 10%. This means that the size of the population will grow by 10% in a month.

- For example, if \( x=1000 \), then the increase in number of the species over the next month will be 10% of that, \( 0.1(1000)=100 \).
- If instead, there were only \( x=100 \) of the species, then at the end of the month there would only be \( 0.1(100)=10 \) more.

For the queen conch, the growth rate of the population depends on the size of the population.

- If there are fewer than 50 queen conchs in a hectare (1000 meters by 1000 meters), then the population growth rate will be relatively low because the adult conchs will have problems finding mates. Queen conchs don’t move too fast. Let’s suppose that if there were only 30 adult conchs, the population growth rate might be around 10%.
- If the number of queen conchs is more than 50 in a hectare, then the growth rate will be larger. If there were 100 queen conchs in a hectare, the growth rate might be 20%.
- If there are too many conchs in the area, then the growth rate will be small again, because of a lack of sufficient food for the entire population. For example, if there were 170 adult conchs, the growth rate might be back down to 10%.

For species such as the queen conch, we can represent the growth rate using a quadratic function. We will use the equation

\[
r = -0.002x^2 + 0.4x
\]

to estimate the growth rate, \( r \), of the queen conch. If the population size is 100 queen conchs, you can substitute \( x=100 \) conchs to find that the population grows at a rate of \( r=20 \) percent. When \( x=30 \), substitution gives a growth rate of \( r=10.2 \) percent. We do not expect these percents to be exactly right, but they give us a sense of what is happening.

1. Put the growth rate function \( r = -0.002x^2 + 0.4x \) into your calculator and use your TABLE (try TblStart=0; ΔTbl=25) to see the growth rates for different size populations. Notice that when \( x=0 \), the growth rate is 0. This makes sense. If there are no queen conchs, they will not reproduce. Notice that when \( x=200 \), the growth rate is again 0. We say that 200 is the “carrying capacity” of the area. What this means is that if there are 200 conchs in this area, then there will be no further growth in population size, so the number of conchs will remain at 200.

2. Choose a useful WINDOW and look at a graph of \( r = -0.002x^2 + 0.4x \). The graph of the growth rate function shows that as the population size \( x \) increases, the rate it grows increases at first, but when the population size exceeds 100, the growth rate decreases. Sketch the graph, showing the x-intercepts and the point where \( x=100 \).

3. Now consider what happens when conch are fished. Suppose each month fishermen catch 10% of the conch in an area. This means that the population growth rate for the conchs will be reduced by 10%, thus, the growth rate will be \( r = -0.002x^2 + 0.4x - 10 \). What size populations \( x \) result in a 0 growth rate?
To answer, we need the solutions to the equation \(-0.002x^2 + 0.4x - 10 = 0\).

Find the solutions to this equation either by using a table or a graph of
\[ r = -0.002x^2 + 0.4x - 10 \]
on your calculator. Find the solutions to the nearest integer. Note that if the population of conch is less than the smaller solution, the growth rate will be negative causing the conch to eventually die out in this area. The larger solution will be the new ‘carrying capacity’, the number of queen conchs this location can sustain if the fishermen take 10% of them every month.

Around St. Thomas, the average abundance of conch is 17 per hectare. Would you be worried about catching 10% of the conchs if you were interested in having conch when your children grow up? The average around St. Croix is higher than around St. Thomas, but the numbers are very small there, too. (The number of conchs per hectare ranges from 0 to 70 at different sites in the waters around both islands.)

The growth rate function that we gave is a guess. A correct growth rate function for conch will depend on conditions in each particular area since, for example, food will be more abundant in some areas than others. But the general principle we studied does apply to the queen conch.

Note: Thanks to Dr. Barbara Kojis, VI Fish and Wildlife Office, for information about the queen conch populations in the Virgin Islands.
Introduction: This investigation of population growth rates as a function of population size does not address the development of the function. It uses the quadratic model to illustrate a minimum sustainable population size and carrying capacity. It also touches lightly on the meaning of the rise and fall of the function values and the meaning of the negative function values. The investigation is intended to give students an opportunity to work with a function where the best means of locating the zeros is via technology, graphically or numerically.

Answers and teaching suggestions:

1. Put the growth rate function \( r = -0.002x^2 + 0.4x \) into your calculator and use your TABLE (try TblStart=0; \( \Delta \)Tbl=25) to see the growth rates for different size populations. Notice that when \( x=0 \), the growth rate is 0. This makes sense. If there are no queen conchs, they will not reproduce. Notice that when \( x=200 \), the growth rate is again 0. We say that 200 is the “carrying capacity” of the area. What this means is that if there are 200 conchs in this area, then there will be no further growth in population size, so the number of conchs will remain at 200.

There is no need for the student to write anything in response to this step of the activity.

2. Choose a useful WINDOW and look at a graph of \( r = -0.002x^2 + 0.4x \). The graph of the growth rate function shows that as the population size \( x \) increases, the rate it grows increases at first, but when the population size exceeds 100, the growth rate decreases. Sketch the graph, showing the x-intercepts and the point where \( x=100 \).

Discussion of this graph might include the following points:

- When the population is 0, the growth rate is zero because there are no conch to reproduce.
- When the population is 200, the growth rate is zero because that is the carrying capacity of the region. Either space or food limits the population’s further growth.
- If the population does rise above 200, the growth rate becomes negative, which will cause the population to return to a size of 200 and the 0 growth rate.
- As the population increases from 0 to 100, the rate of growth increases, too. That is the population grows faster.
• If the population exceeds 100 conch, the population continues to increase, but the rate of increase slows down until it reaches the carrying capacity of the area.

3. … each month fishermen catch 10% of the conch in an area. This means that the population growth rate for the conchs will be reduced by 10%, thus, the growth rate will be

\[ r = -0.002x^2 + 0.4x - 10 \]

What size populations \( x \) result in a 0 growth rate?

To answer, we need the solutions to the equation

\[ -0.002x^2 + 0.4x - 10 = 0 \]

Find the solutions to this equation either by using a table or a graph of \( r = -0.002x^2 + 0.4x - 10 \) on your calculator. Find the solutions to the nearest integer. Note that if the population of conch is less than the smaller solution, the growth rate will be negative causing the conch to eventually die out in this area. The larger solution will be the new ‘carrying capacity’, the number of queen conchs this location can sustain if the fishermen take 10% of them every month.

The solutions are (approximately) \( x=29 \) and \( x=171 \). So the minimum sustainable population in the area is 29. If there are fewer than 29, the population growth rate is negative and the population dies out. The carrying capacity is 171. If there are more than 171 conch, the area will have more conch than it can sustain and the growth rate will be negative, taking the population back to 171, the carrying capacity. For any population between 29 and 171, the growth rate is positive, moving the population toward 171.

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Note that with our model we do not know the size of the area. However, 17 per acre is a dangerously low number for sustainability. The Virgin Islands has strict limits on conch take and strict seasons when conch may be taken. Getting local fishermen and local restaurants to respect these rules will require education. Mathematics can help.
The growth rate function that we gave is a guess. A correct growth rate function for conch is not known; such a function will depend on conditions in each particular area since food will be more abundant in some areas than others. But the general principle we studied does apply to the queen conch.

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