Air Pollution

Have you ever been on Main Street in Charlotte Amalie on a hot day when there was an enormous amount of traffic on the street and the air you were breathing felt dirty? In large urban areas and in industrial areas, air pollution increases during the day as auto emissions and industrial pollution enter the atmosphere.

Last year on July 10th, the pollution in one city was 20 parts per million (ppm) at 8 a.m., and it increased by 16 ppm each hour through the day.

1. Complete this table:

<table>
<thead>
<tr>
<th>x hours after 8 a.m.</th>
<th>y pollution in ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

2. Were you able to get an expression in terms of x to put in the y-column opposite “x”? Here is a way to think through that part of the question.

We know that

y depends on x
y is 20 when x = 0
y is 20 + 16 when x = 1
y is 20 + two 16s when x = 2; thus y is 20 + 2·16 when x = 2
y is 20 + 3·16 when x = 3
y is 20 + ___·16 when x = 4 (fill in the blank)

We can generalize this: y is 20 with another 16 added to it for each hour.
If there were x hours, that would be ____ + ____ + ____
or, written with style(!) __________

Summary: The pollution was ___ when x was 0. ___ more was added each time x increased by 1 more hour. Thus, y = _______
3. The Air Quality Management department of an urban government is usually required to publish an Unhealthy Air Alert any day that the air pollution reaches 150 ppm. If the mathematical model we developed is valid from 8 in the morning until 6 in the evening on that day, was it necessary to publish an unhealthy air alert on that day?

4. Write an equation saying that the amount of air pollution at time x is 150 ppm. (See question 2)

5. Solve the equation you wrote in question 4.

6. What does the solution to your equation mean in terms of the situation?

7. Put the equation you wrote in question 2 on the Y= screen of your calculator. Use the table to see how long it would take for pollution to exceed 400 ppm if it continued to increase at the same rate.

8. Use a graph to determine when pollution was 124 ppm. Explain how you do it.

9. Sometimes there is less air pollution than other times. On the night of July 10th, the day we’ve been studying in this lesson, it rained in the city; the rain cleaned the atmosphere. Of course, the next day, industry and automobiles were back at it. Nevertheless, the pollution was only half of what it had been the previous day at the same hours.

   How much pollution was in the air (in ppm) at 8 am on July 11?
   How much was added each hour (in ppm)?
   How much pollution was in the air x hours after 8 am on July 11?
   Compare this with your answer for x hours after 8 am on July 10.

   Notice that \( \frac{1}{2} (20 + 16x) = 10 + 8x \). This is an example of the distributive law.

Homework

1. Simplify \( \frac{1}{2} (4x + 2) - \frac{1}{3} (6x - 3) \)
2. Simplify \( 0.01(200x - 4000) \)
3. Find 25% of \( (8000x + 100) \)
4. Simplify \( 2(x + 9) + 3(x - 5) \). Do you have 5x + 3 for the simplified expression? We say these two expressions are equivalent because for every input value of x, both of them would give the same output. You can see this graphically and numerically.

   Enter \( Y_1 = \) original expression
   \( Y_2 = \) simplified expression.

   Then use TRACE and the ↑ and ↓ arrow keys to see that the two expressions give exactly the same output for any x you try.

   Also go to the table. How can you tell from the table that the two expressions are equivalent?
Teaching Guide for

Air Pollution

Introduction: The mathematics in this lesson includes translating a verbal description into numerical information, generalizing a table of values to get a linear expression, setting up and solving an equation, and an introduction to the distributive property of multiplication over addition.

Although students rarely have any difficulty completing the numerical part of the table in the Air Pollution lesson, it is a rare student at the level of these lessons who can easily generalize the thinking to get an algebraic expression for \( y \) after \( x \) hours. Question 2 is intended to show the students how to think through the problem systematically so that the equation for \( x \) hours becomes “obvious”. What we would like to get here is a series of “Aha!”s from around our classrooms, or at least a widespread “Oh, I see.”

The additional “homework” problems should provide valuable material for discussion in the class following the homework. This is an opportunity not just for practicing the distributive property of multiplication over addition but also to review the meaning of the fractions, decimals, and percents and to gain more insight into the meaning of the equality of two algebraic expressions.

Answers and teaching suggestions:

1. | \( x \) hours after 8 a.m. | \( y \) pollution in ppm |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>68</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
</tr>
<tr>
<td>( x )</td>
<td>( 20 + 16x )</td>
</tr>
</tbody>
</table>

2. \( y \) is \( 20 + 4 \cdot 16 \) when \( x = 4 \) (fill in the blank)
   We can generalize this: \( y \) is 20 with another 16 added to it for each hour.
   If there were \( x \) hours, that would be \( 20 + 16 \cdot x \)
   or, written with style(!) \( 20 + 16 \cdot x \)
   Summary: The pollution was 20 when \( x \) was 0. 16 more was added each time \( x \) increased by 1 more hour. Thus, \( y = 20 + 16x \)
   When all groups have completed question 2, it is valuable to bring students together into a short whole-class discussion. Get someone to describe the thinking that was used to get the result, and assure that every student has a good understanding. Please point out during this discussion that the number 16 in this situation where \( y = 20 + 16x \) gives us another example of a rate; the pollution is increasing at a rate of 16 ppm/hour.

3. Some students may answer this by determining whether pollution will have reached 150 ppm by 6 pm, when \( x = 10 \). Since \( 20 + 16 \cdot 10 = 180 > 150 \), it will be necessary to publish an alert. Others may solve the equation \( 20 + 16x = 150 \). This results in \( x = \frac{81}{8} \). It may still be a
challenge to determine that this means that an alert will have to be published. Still others may simply continue the table until y exceeds 150. That provides them an introduction to question 4.

4. $20 + 16x = 150$. $x = \frac{8}{5}$.

5. $x = 8\frac{1}{5}$.

6. It means that when $x = 8\frac{1}{5}$, that is, $8\frac{1}{5}$ hours after 8 am, just after 4 pm, the city will need to publish an air pollution alert because the level of air pollution will have reached the danger level of 150 ppm.

7. Input $Y=20 + 16x$. Set TableStart = 0 (or 8 or 10, or other starting point depending on student insight), and $\Delta Tbl = 1$. Go to table and search for 400 in the Y column. Students will see

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>288</td>
</tr>
<tr>
<td>24</td>
<td>404</td>
</tr>
</tbody>
</table>

This means that if pollution continued to rise at the same rate (16 ppm/hour), it would exceed 400 ppm some time between 23 and 24 hours after 8am on July 10; that is, at about 8am on July 11.

8. $x = 6.5$. Be sure that students interpret this result as 6.5 hours after 8:00, thus 2:30pm. Students may use several methods of exploring the graph on a graphing calculator. Encourage them to share their methods and insights with each other; be sure that every student understands and can use at least one approach effectively. For example, one student might graph $y_1 = 20 + 16x$ and $y_2 = 124$ with a window such as [0, 9.4] for x and [0, 150] for y, and find the intersection point by tracing. Another student might graph only $y_1 = 20 + 16x$ and trace until she sees a y-value of 124. (Again, the friendly window [0, 9.4] for x would facilitate the search.) Yet another student might graph both $y_1 = 20 + 16x$ and $y_2 = 124$ with no particular concern for the friendly window and use 2ndCALC INTERSECT to get the coordinates of the point.

9. At 8 am on July 11, the level of pollution was half as much as the day before, so 10 ppm. 8ppm were added each hour. 10 + 8x will give the level of pollution (in ppm) in the air x hours after 8 am on July 11. Comparing this with the expression for x hours after 8 am on July 10 we see $10 + 8x$ compared with $20 + 16x$. This suggests $\frac{1}{2}(20 + 16x) = 10 + 8x$, introducing the distributive property of multiplication over addition. Be sure students note this and fully take it in.

Answers to homework exercises.

1. $\frac{1}{2}(4x + 2) - \frac{1}{3}(6x - 3) = 2x + 1 - 2x + 1 = 2$

2. $0.1(200x - 4000) = 2x - 40$

3. 25% of (8000x + 100) is 2000x + 25

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4. Students can verify that \(2(x + 9) + 3(x - 5)\) is equivalent to \(5x + 3\) graphically by using a graphing calculator as described in the problem. They can verify it numerically by looking at the table and seeing that regardless of the value of \(x\), the \(y\) values are always identical.